

M2 to D2 and vice versa by 3-Lie and Lie bialgebra

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Abstract

Using the concept of 3-Lie bialgebra, which has recently been defined in arXiv:1604.04475, we construct Bagger-Lambert-Gustavson (BLG) model for M2-brane on Manin triple of a special 3-Lie bialgebra. Then by using of correspondence and relation between those 3-Lie bialgebra with Lie bialgebra, we reduce this model to an $N = (4, 4)$ WZW model (D2-brane), such that, its algebraic structure is a Lie bialgebra with one 2-cocycle. In this manner by using correspondence of 3-Lie bialgebra and Lie bialgebra (for this special 3-Lie algebra) one can construct M2-brane from a D2-brane and vice versa.

Keywords: String theory, M-theory, Lie bialgebra, 3-Lie bialgebra, Manin triple.

1 Introduction

M-theory is a magical theory with quite little knowledge about it, such that, to improve its current definition we must focus on 11 dimensional (11d) supergravity [1]. These supergravity theories are low-energy limit of M-theory, whereas, 10d supergravity is the low-energy limit of superstring. Therefore, there should be a connection between M-theory and string theory like existing connection between 11d and 10d supergravity. If we could make this connection clear, then a lot of unknown issues about M-theory would be resolved. Our knowledge about M-theory has been achieved by compare and grope with D-branes [2]. Many attempts have been made to obtain effective action for multiple M2-brane, the most important of which are cited in [3–8]. Basu and Harvey [9] applied 3-bracket in BPS equation in order to explain N coincided M2-branes ending on M5-brane. This equation has been given by comprising Nahm equation for string theory [10]. Bagger and Lambert [3–5] and Gustavsson [6] independently write transformation of fields for M2-brane according to D2-brane. They obtain equation of motion of fields by using closure of supersymmetric transformation algebra, and writing a Lagrangian in a way that its equations of motion are the same. Bagger-Lambert-Gustavsson (BLG) model [3–6] has a Lagrangian with maximal supersymmetry ($N = 8$) for description of two M2-branes [11], which use 3-Lie algebra. As the earlier example, the algebra A_4 was the only known non-trivial 3-Lie algebra [5]. Mukhi and Papageorgakis [12] were able to convert the topological term (Chern-Simon) to the dynamical one, i.e., Yang-Mills term, by assigning a vacuum expectation value to a scalar fields of BLG lagrangian and using Higgs mechanism. On the other hand, if one dualizes M-theory on a circle, one can obtain type IIA string theory (D2-brane) [13]. This can be considered as a trick for going from M2 to D2, as it was performed for A_4 . Consequently, a 3-Lie algebra was constructed from arbitrary Lie algebra [14] and BLG model on this 3-Lie algebras were studied in later works [15–19]. In all these works one can obtain M2-model to D2 but these are not standard methods of construction of M2 from D2. Here we will try to perform another method using the concept of 3-Lie bialgebra [20].

Lie bialgebras [21] are algebraic structures of Poisson-Lie groups [22] which play an important role in the theory of classical integrable systems (see [23] for a review). They also play an important role in $N = (2, 2)$ and $N = (4, 4)$ supersymmetric WZW models [24, 25]. In Ref. [26] we have studied the algebraic structure of $N = (2, 2)$ and $N = (4, 4)$ supersymmetric WZW models in more detail. The concept of 3-Lie algebra was described in Fillipov’s work for the first time [27] following the pioneering work of Nambu in different formulation

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of classical mechanics [28]. In Ref. [20] we define the concept of 3-Lie bialgebra by using cohomology of 3-Lie algebras. We believe that introduction of 3-Lie bialgebras can play an important role in M-theory. In this paper we will follow some steps in this direction. We will express the BLG Lagrangian on the Manin triple of a especial 3-Lie bialgebra and use similar procedure applied by Mukhi and Papageorgakis [12], in obtaining the Yang-Mills in addition to other terms which are that of WZW. The extra term is square of B-field for WZW models [29]. If the space-time coordinates were algebraic indices (like space-time coordinates ,i.e., scalar and fermion fields in BLG model) then B-field of $N = (4, 4)$ WZW model could be obtained from this form. In this way if one knows about $N = (4, 4)$ WZW models in detail, then one can obtain information about BLG and vice versa, i.e., one can construct M2 from D2-model and vice versa.

The outline of the paper is as follows. We review BLG action and the correspondence between M-theory and string theory in section two. In section three we review the definition of 3-Lie bialgebra [20] and give an example ,such that, is a one-to-one correspondence between 3-Lie bialgebra and Lie bialgebra. Consequently, in section four we express BLG model (M2-model) on the Manin triple of that 3-Lie bialgebra and show that it turns into Yang-Mills and $N = (4, 4)$ WZW model (D2-model). In this manner we show that using the correspondence of 3-Lie bialgebra and Lie bialgebra one can construct M2-model from D2 and vice versa. For further description of the obtained model; in the section five we show that the WZW model can be obtained from a DBI action with extra Lie algebra valued fields.

2 BLG model

Here for self consistency of the paper and presentation of the notation, we give a short review of BLG model. The multiple M2-brane model of Bagger-Lambert [3–5] and Gustavsson [6] (BLG) is based on 3-Lie algebra. In this algebra the Lie bracket is generalized to 3-Lie bracket [27]. n-Lie algebra was introduced by Filippov in 1985 [27] as an extension of the Nambu bracket [28] to Lie algebras. 3-Lie algebras are a special kind of n-Lie algebras, and have many applications in mathematical and theoretical physics [3–6]. The 3-Lie algebra \mathcal{A} [27], [30] with the basis $\{T^a\}$ is a vector space \mathcal{A} which endowed with the following three antisymmetric bracket:

$$[T^a, T^b, T^c] = f^{abc} T^d, \quad a, b, c, d = 1, \dots, \dim \mathcal{A}, \quad (1)$$

such as, to satisfy the following fundamental identity [27]:

$$[T^g, T^d, [T^a, T^b, T^c]] = [[T^g, T^d, T^a], T^b, T^c] + [T^a, [T^g, T^d, T^b], T^c] + [T^a, T^b, [T^g, T^d, T^c]], \quad (2)$$

where it can be redefined by structure constant of \mathcal{A} (f_d^{abc}) in the following form:

$$f^{abc} f^{gde} f_f - f^{gda} f^{ebc} f_f - f^{gdb} f^{aec} f_f - f^{gdc} f^{abe} f_f = 0. \quad (3)$$

For the BLG model the following supersymmetric (SUSY) transformations are proposed [3–5], as

$$\begin{aligned} \delta X_a^I &= i\bar{\epsilon} \Gamma^I \Psi_a, \\ \delta \Psi_a &= D_\mu X_a^I \Gamma^\mu \Gamma_I \epsilon - \frac{1}{2} X_b^I X_c^J X_d^K f^{bcd}{}_a \Gamma_{IJK} \epsilon, \\ \delta(\hat{A}_\mu)_b^a &= i\bar{\epsilon} \Gamma_\mu \Gamma_I X_c^I \Psi_d f^{cda}{}_b, \end{aligned} \quad (4)$$

where $(\hat{A}_\nu)_a^b = f^{cdb}{}_a A_{\nu cd}$, and indices $I, J, \dots = 1, 2, \dots, 8$ apply for transverse coordinates with $SO(8)$ symmetry (R-symmetry); and indices $\mu, \nu, \dots = 0, 1, 2$ indicate the world volume coordinate with symmetry $SO(1, 2)$ ¹. Also, Γ_I s are Dirac matrices and X^I are the transverse coordinates of 3-Lie algebra valued coordinates and Ψ is a 16 component Majorana spinor of 3-Lie algebra valued, conforming with chirality condition using the following relation:

$$\Gamma^{012} \Psi = -\Psi, \quad (5)$$

such that, for supersymmetric parameter ϵ we have:

$$\Gamma^{012} \epsilon = \epsilon, \quad (6)$$

¹Presence of M2-brane breaks Lorentz invariance $SO(1, 10)$ to $SO(1, 2) \times SO(8)$.

and the covariant derivative D_μ has the form :

$$D_\mu X_a^{(I)} = \partial_\mu X_a^{(I)} + f^{bcd}{}_a A_{\mu cd} X_b^{(I)}. \quad (7)$$

Furthermore, the Γ_{IJ} and Γ_{IJK} have the following forms:

$$\Gamma^{IJ} = \frac{1}{2}(\Gamma^I \Gamma^J - \Gamma^J \Gamma^I), \quad (8)$$

$$\{\Gamma^I, \Gamma_{JKL}\} = 6\delta^I_{[J} \Gamma_{KL]}. \quad (9)$$

Using the assumption that the algebra of SUSY transformations (4) must be closed, the following relations can be realized [3]:

$$\Gamma^\mu D_\mu \Psi_a + \frac{1}{2} \Gamma_{IJ} X_c^I X_d^J \Psi_b f^{cdb}{}_a = 0, \quad (10)$$

$$D^2 X_a^I - \frac{i}{2} \tilde{\Psi}_c \Gamma_J^I X_d^J \Psi_b f^{cdb}{}_a + \frac{1}{2} f^{bcd}{}_a f^{efg}{}_d X_b^J X_c^K X_e^I X_f^J X_g^K = 0, \quad (11)$$

$$(\hat{F}_{\mu\nu})_a^b + \epsilon_{\mu\nu\lambda} (X_c^J D^\lambda X_d^J + \frac{i}{2} \tilde{\Psi}_c \Gamma^\lambda \Psi_d) f^{cdb}{}_a = 0, \quad (12)$$

where,

$$(\hat{F}_{\mu\nu})_a^b = \partial_\mu (\hat{A}_\nu)_a^b - \partial_\nu (\hat{A}_\mu)_a^b + (\hat{A}_\mu)_c^b (\hat{A}_\nu)_a^c - (\hat{A}_\nu)_c^b (\hat{A}_\mu)_a^c, \quad (13)$$

and $D^2 = D_\mu D^\mu$. Similarly Bagger and Lambert have proposed the following Lagrangian [4] such that the relations (10)-(12) are its equations of motion:

$$\begin{aligned} L = & - \frac{1}{2} D_\mu X^{a(I)} D^\mu X_a^{(I)} + \frac{i}{2} \bar{\psi}^a \Gamma^\mu D_\mu \psi_a + \frac{i}{4} f^{abcd} \bar{\psi}_b \Gamma^{IJ} X_{c(I)} X_{d(J)} \psi_a \\ & - \frac{1}{12} f^{abcd} f^{efg}{}_d X_a^{(I)} X_b^{(J)} X_c^{(K)} X_e^{(I)} X_f^{(J)} X_g^{(K)} \\ & + \frac{1}{2} \epsilon^{\mu\nu\lambda} [f^{abcd} A_{\mu ab} \partial_\nu A_{\lambda cd} + \frac{2}{3} f^{cda}{}_g f^{efgb} A_{\mu ab} A_{\nu cd} A_{\lambda ef}]. \end{aligned} \quad (14)$$

The above Lagrangian is invariant under SUSY transformation (4). In order for the degrees of Fermion and Boson not to vary in Lagrangian, one must use topological term which is the Chern-Simon term (the fifth term in the bracket at the above Lagrangian) [31]:

$$L_{CS} = Tr(\epsilon^{\mu\nu\lambda} (A_\mu \partial_\nu A_\lambda + \frac{2i}{3} (A_\mu A_\nu A_\lambda))), \quad (15)$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu], \quad (16)$$

which equipped with 3-Lie algebra. Now, we will try to extract the BLG model from a superstring model (D2) and vice versa. In our perspective for this propose we need to apply the concept of 3-Lie bialgebra. The definition of 3-Lie bialgebra is given in [20], however, for self containment of the paper we give a short review of this concept in the following section.

3 3-Lie bialgebra

In this section we review the definitions of 3-Lie bialgebra.

Definition: A Lie algebra \mathcal{G} with co commutator $\delta : \mathcal{G} \rightarrow \mathcal{G} \otimes \mathcal{G}$ is a Lie bialgebra if [23]:
 $a) \delta$ is a one-cocycle, i.e.:

$$\delta([T^i, T^j]) = ad^{(2)}_{T^j} \delta(T^i) - ad^{(2)}_{T^i} \delta(T^j), \quad (17)$$

where

$$ad^{(2)}_{T^j} = ad_{T^j} \otimes 1 + 1 \otimes ad_{T^j}, \quad (18)$$

and $\{T^i\}$ s are bases for the Lie algebra \mathcal{G} , (here 1 is an identity map on \mathcal{G}),

b) the dual map ${}^t\delta : \mathcal{G}^* \otimes \mathcal{G}^* \rightarrow \mathcal{G}^*$ is a commutator on \mathcal{G}^* (dual space of \mathcal{G}) as the following definition:

$$(\tilde{T}_i \otimes \tilde{T}_j, \delta(T^k)) = ({}^t\delta(\tilde{T}_i \otimes \tilde{T}_j), T^k) = ([\tilde{T}_i, \tilde{T}_j], T^k), \quad (19)$$

where $\{\tilde{T}_i\}$ is the base for the space \mathcal{G}^* and $(,)$ is the pairing between \mathcal{G} and \mathcal{G}^* . In this way there is a Lie algebra structure on the space \mathcal{G}^* . The Lie bialgebra is shown with (\mathcal{G}, δ) or $(\mathcal{G}, \mathcal{G}^*)$.

Definition: $(\mathcal{D}, \mathcal{G}, \mathcal{G}^*)$ is a Manin triple, the triple of Lie algebras \mathcal{D} , \mathcal{G} and \mathcal{G}^* , such that, there is a nondegenerate, symmetric and ad-invariant inner product on \mathcal{D} with the following properties [23]:

- a) \mathcal{G} and \mathcal{G}^* are subalgebras of \mathcal{D} .
- b) $\mathcal{D} = \mathcal{G} \oplus \mathcal{G}^*$ as a vector space.
- c) \mathcal{G} and \mathcal{G}^* are isotropic, i.e.,

$$(T^i, \tilde{T}_j) = \delta_j^i, \quad (T^i, T^j) = (\tilde{T}_i, \tilde{T}_j) = 0.$$

The Jacobi identity for $\mathcal{D} = \mathcal{G} \oplus \mathcal{G}^*$ results in the following identities [32]:

$$f^{ij}_k f^{kl}_m - f^{ik}_m f^{jl}_k + f^{jk}_m f^{il}_k = 0, \quad (20)$$

$$\tilde{f}_{ij}^k \tilde{f}_{kl}^m - \tilde{f}_{ik}^m \tilde{f}_{jl}^k + \tilde{f}_{jk}^m \tilde{f}_{il}^k = 0, \quad (21)$$

$$-f^{ij}_k \tilde{f}_{lm}^k + f^{ik}_l \tilde{f}_{km}^j - f^{jk}_m \tilde{f}_{lk}^i - f^{jk}_l \tilde{f}_{km}^i + f^{ik}_m \tilde{f}_{lk}^j = 0, \quad (22)$$

where f^{ij}_k and \tilde{f}_{ij}^k are the structure constants of the Lie algebras \mathcal{G} and \mathcal{G}^* , respectively (i.e. $[T^i, T^j] = f^{ij}_k T^k$, $[\tilde{T}_i, \tilde{T}_j] = \tilde{f}_{ij}^k \tilde{T}_k$). Note that (20) and (21) are Jacobi identities for the Lie algebras \mathcal{G} and \mathcal{G}^* , respectively, and (22) is the mix Jacobi identity on \mathcal{D} .

Theorem: *There exist a one to one correspondence between Lie bialgebra $(\mathcal{G}, \mathcal{G}^*)$ and Manin triple $(\mathcal{D}, \mathcal{G}, \mathcal{G}^*)$ [23].*

We have defined the 3-Lie bialgebra in [20] as follows:

Definition: A 3-Lie algebra \mathcal{A} with co commutator $\delta : \mathcal{A} \rightarrow \mathcal{A} \otimes \mathcal{A} \otimes \mathcal{A}$ is a 3-Lie bialgebra if [20]:

a) δ is a one-cocycle of \mathcal{A} with value in $\otimes^3 \mathcal{A}$, i.e:

$$\delta([T^a, T^b, T^c]) = ad^{(3)}_{T^b \otimes T^c} \delta(T^a) - ad^{(3)}_{T^a \otimes T^c} \delta(T^b) + ad^{(3)}_{T^a \otimes T^b} \delta(T^c), \quad (23)$$

such that,

$$ad^{(3)}_{T^b \otimes T^c} = ad_{T^b \otimes T^c} \otimes 1 \otimes 1 + 1 \otimes ad_{T^b \otimes T^c} \otimes 1 + 1 \otimes 1 \otimes ad_{T^b \otimes T^c}, \quad (24)$$

where $\{T^a\}$ s are bases of 3-Lie algebra \mathcal{A} and we have $ad_{T^a \otimes T^b} T^c = [T^a, T^b, T^c]$ [33].

b) the dual map ${}^t\delta : \otimes^3 \mathcal{A}^* \rightarrow \mathcal{A}^*$ is a 3-Lie bracket on \mathcal{A}^* (dual space of \mathcal{A}) which is a commutator on \mathcal{G}^* satisfying the fundamental identity :

$$(\tilde{T}_a \otimes \tilde{T}_b \otimes \tilde{T}_c, \delta(T^d)) = ({}^t\delta(\tilde{T}_a \otimes \tilde{T}_b \otimes \tilde{T}_c), T^d) = ([\tilde{T}_a, \tilde{T}_b, \tilde{T}_c], T^d), \quad (25)$$

in which $\{\tilde{T}_a\}$ is the base for the space \mathcal{A}^* and $(,)$ is a natural pairing between \mathcal{A} and \mathcal{A}^* . In this way \mathcal{A}^* constructs a 3-Lie algebra. The 3-Lie bialgebra can be denoted either by $(\mathcal{A}, \mathcal{A}^*)$ or (\mathcal{A}, δ) .

Definition: $(\mathcal{D}, \mathcal{A}, \mathcal{A}^*)$ is Manin triple, a triple of 3-Lie algebras \mathcal{D} , \mathcal{A} and \mathcal{A}^* such that there is a nondegenerate, symmetric and ad-invariant inner product on \mathcal{D} with the following properties [20]: ²

² Note that in general vector space \mathcal{D} is not a 3-Lie algebra.

- a) \mathcal{A} and \mathcal{A}^* are 3-Lie subalgebras of \mathcal{D} ,
- b) $\mathcal{D} = \mathcal{A} \oplus \mathcal{A}^*$ as a vector space,
- c) \mathcal{A} and \mathcal{A}^* are isotropic, i.e.

$$(T^a, \tilde{T}_b) = \delta_b^a, \quad (T^a, T^b) = (\tilde{T}_a, \tilde{T}_b) = 0.$$

By using the fundamental identity (2), equation (23) and relation $\delta(T^a) = \tilde{f}_{bcd}^a T^b \otimes T^c \otimes T^d$, one can obtain the following fundamental and mix fundamental identities [20]:

$$f^{aef}_g f^{bcdg} - f^{bef}_g f^{acdg} + f^{cef}_g f^{abdg} - f^{def}_g f^{abcg} = 0, \quad (26)$$

$$\tilde{f}^{aef}_g \tilde{f}^{bcdg} - \tilde{f}^{bef}_g \tilde{f}^{acdg} + \tilde{f}^{cef}_g \tilde{f}^{abdg} - \tilde{f}^{def}_g \tilde{f}^{abcg} = 0, \quad (27)$$

$$\begin{aligned} f^{abc}_g \tilde{f}^{def}_g &= f^{gbc}_f \tilde{f}^{deg}_g + f^{gbc}_e \tilde{f}^{dfg}_g - f^{gbc}_d \tilde{f}^{efg}_g - f^{gac}_f \tilde{f}^{deg}_g + f^{gac}_e \tilde{f}^{dfg}_g \\ &\quad - f^{gac}_d \tilde{f}^{efg}_g + f^{gab}_f \tilde{f}^{deg}_g - f^{gab}_e \tilde{f}^{dfg}_g + f^{gab}_d \tilde{f}^{efg}_g, \end{aligned} \quad (28)$$

where f^{abc}_d and \tilde{f}^{abc}_d are structure constants of 3-Lie algebras \mathcal{A} and \mathcal{A}^* , respectively.

3.1 An example

Now, we will consider an especial example 3-Lie bialgebra related to 3-Lie algebra $\mathcal{A}_{\mathcal{G}}$ and Lie algebra \mathcal{G}^3 . The 3-Lie algebras $\mathcal{A}_{\mathcal{G}}$ (mentioned in [14] for a first time) have commutation relations as follows:

$$[T^-, T^a, T^b] = 0, \quad [T^+, T^i, T^j] = f^{ij}_k T^k, \quad [T^i, T^j, T^k] = f^{ijk} T^-, \quad (29)$$

where $\{T^i\}$ s are basis of the Lie algebra \mathcal{G} ($[T^i, T^j] = f^{ij}_k T^k$ with $i, j, k = 1, 2, \dots, \dim \mathcal{G}$) and f^{ij}_k is its structure constant⁴. Furthermore, T^- and T^+ are new generators and we have $a = +, -, i$. Now we propose that there exists a 3-Lie algebra structure on $\mathcal{A}_{\mathcal{G}^*}$ with similar commutation relations:

$$[\tilde{T}_-, \tilde{T}_a, \tilde{T}_b] = 0, \quad [\tilde{T}_+, \tilde{T}_i, \tilde{T}_j] = \tilde{f}^{ij}_k \tilde{T}_k, \quad [\tilde{T}_i, \tilde{T}_j, \tilde{T}_k] = \tilde{f}^{ijk} T_-, \quad (30)$$

such that $\mathcal{G}^*([\tilde{T}_i, \tilde{T}_j] = \tilde{f}^{ij}_k \tilde{T}_k$ with $i, j, k = 1, 2, \dots, \dim \mathcal{G}^*)$ is a Lie algebra.

Proposition: [34] A 3-Lie algebra $\mathcal{A}_{\mathcal{G}}$ construct a 3-Lie bialgebra $(\mathcal{A}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}^*)$ if and only if $(\mathcal{G}, \mathcal{G}^*)$ is a Lie bialgebra. The proof can be found in [34].

4 BLG model on Manin of 3-Lie algebras (M2 \leftrightarrow D2)

In the previous section we have considered a especial case of the Manin triple $(\mathcal{D}, \mathcal{A}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}^*)$ and have noted that there is a correspondence between 3-Lie bialgebra $(\mathcal{A}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}^*)$ and Lie bialgebra $(\mathcal{G}, \mathcal{G}^*)$ ⁵. Now we want to apply this 3-Lie algebra \mathcal{D} in the BLG model. We obtained in the previous section that:

$$F^{-AB}_C = 0, \quad F^{\tilde{-}AB}_C = 0, \quad (31)$$

then

$$F^{ABC}_+ = 0, \quad F^{ABC}_{\tilde{+}} = 0, \quad (32)$$

Note that for the Manin triple \mathcal{D} we apply the symbol F^{ABC}_D for the structure constant of the Manin triple as a $(4 + 2\dim \mathcal{G})$ ⁶ dimensional 3-Lie algebra, i.e. we have T^A as a basis for the Manin triple with $A = +, -$.

³Note that this example of 3-Lie bialgebra was considered in [34] as a first time. Here for a self containing of the paper we denote it as an example.

⁴Note that the indices of f^{ij}_k are lowered and raised by the ad-invariant metric g^{ij} of the Lie algebra \mathcal{G} .

⁵In general the vector space \mathcal{D} in triple $(\mathcal{D}, \mathcal{A}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}^*)$ is not 3-Lie algebra and also there is not correspondence between Manin triple $(\mathcal{D}, \mathcal{A}_{\mathcal{G}}, \mathcal{A}_{\mathcal{G}}^*)$ and Lie bialgebra $(\mathcal{G}, \mathcal{G}^*)$, but for the above special example \mathcal{D} is a 3-Lie algebra and for this case there is correspondence.

⁶We assume \mathcal{G} and \mathcal{G}^* have same dimension i.e. $2 + \dim \mathcal{G}$ that 2 is used for $+$ and $-$.

$T^+ = T^+$; $A = -$, $T^- = T^-$; $A = i$, $T^i = T^i$; $A = i + (2 + \dim \mathcal{G})$, $T^{i+(2+\dim \mathcal{G})} = T^{\tilde{i}}$; $A = (-) + (2 + \dim \mathcal{G})$, $T^{(-)+(2+\dim \mathcal{G})} = T^{\tilde{-}}$ and $A = (+) + (2 + \dim \mathcal{G})$, $T^{(+)+(2+\dim \mathcal{G})} = T^{\tilde{+}}$ together with the following commutation relations:

$$\begin{aligned} [T^-, T^A, T^B] &= 0, & [T^+, T^i, T^j] &= f^{ij}{}_k T^k, & [T^+, T^i, T^{\tilde{j}}] &= f^{ik}{}_j T^{\tilde{k}}, & [T^i, T^j, T^k] &= f^{ijk} T^-, \\ [T^{\tilde{-}}, T^A, T^B] &= 0, & [T^{\tilde{+}}, T^{\tilde{i}}, T^{\tilde{j}}] &= \tilde{f}_{ij}{}^k T_{\tilde{k}}, & [T^{\tilde{+}}, T^{\tilde{i}}, T^j] &= \tilde{f}_{ik}{}^j T^k, & [T^{\tilde{i}}, T^{\tilde{j}}, T^{\tilde{k}}] &= \tilde{f}_{ijk} T_{\tilde{-}}, \\ [T^{\tilde{k}}, T^i, T^j] &= f^{ji}{}_k T^{\tilde{-}}, & [T^{\tilde{+}}, T^j, T^k] &= f^{ijk} T^{\tilde{i}}, & [T^+, T^{\tilde{j}}, T^{\tilde{k}}] &= \tilde{f}_{ijk} T^i, & [T^k, T^{\tilde{i}}, T^{\tilde{j}}] &= -\tilde{f}_{ij}{}^k T^-. \end{aligned} \quad (33)$$

Now we write the equations of motion for the BLG model (10)-(12) by considering 3-Lie algebra \mathcal{D} of the Manin triple related to this especial 3-Lie bialgebra as follows:

$$\Gamma^\mu D_\mu \Psi_A + \frac{1}{2} \Gamma_{IJ} X_C^I X_D^J \Psi_B F^{CDB}{}_A = 0, \quad (34)$$

$$D^2 X_A^I - \frac{i}{2} \tilde{\Psi}_C \Gamma_J^I X_D^J \Psi_B F^{CDB}{}_A + \frac{1}{2} F^{BCD}{}_A F^{EFG}{}_D X_B^J X_C^K X_E^I X_F^J X_G^K = 0, \quad (35)$$

$$(\hat{F}_{\mu\nu})_A^B + \epsilon_{\mu\nu\lambda} (X_C^J D^\lambda X_D^J + \frac{i}{2} \tilde{\Psi}_C \Gamma^\lambda \Psi_D) F^{CDB}{}_A = 0, \quad (36)$$

such that, if we take $A = +, \tilde{+}$ in (34) and (35), then we obtain the following relations:

$$\partial^2 X_+^I = 0, \quad \Gamma^\mu \partial_\mu \Psi_+ = 0, \quad \partial^2 X_{\tilde{+}}^I = 0, \quad \Gamma^\mu \partial_\mu \Psi_{\tilde{+}} = 0, \quad (37)$$

this means that $(X_+^I, X_{\tilde{+}}^I)$ and $(\Psi_+, \Psi_{\tilde{+}})$ can be set to a constant such as Yang-Mills coupling [13] as applied to the whole theory and zero, respectively. This constant must conserve the SUSY transformations (4) for Manin triple. Then, from the following relations:

$$\delta X_+^I = i\epsilon \Gamma^I \Psi_+, \quad (38)$$

$$\delta \Psi_+ = \partial_\mu X_+^I \Gamma^\mu \Gamma^I \epsilon, \quad (39)$$

one can show that SUSY transformations do not change if we assign a vacuum expectation value (VEV) to one of the fields. Then, the Lagrangian terms become as follows: by considering this relations:

$$D_\mu X_-^{(I)} = \partial_\mu X_-^{(I)} + f^{ijk} A_{\mu jk} X_i^{(I)} + 2\tilde{f}_{ji}{}^k A_{\mu \tilde{j}k} X_{\tilde{i}}^{(I)} + \tilde{f}_{kj}{}^i A_{\mu \tilde{j}\tilde{k}} X_i^{(I)}, \quad (40)$$

$$D_\mu X_-^{(I)} = \partial_\mu X_-^{(I)} + \tilde{f}_{ijk} A_{\mu \tilde{j}\tilde{k}} X_{\tilde{i}}^{(I)} + f^{jk}{}_i A_{\mu jk} X_i^{(I)} + 2f^{ij}{}_k A_{\mu \tilde{j}\tilde{k}} X_i^{(I)}, \quad (41)$$

$$\begin{aligned} D_\mu X_i^{(I)} &= \partial_\mu X_i^{(I)} + f^{jk}{}_i A_{\mu jk} X_+^{(I)} + \tilde{f}_{ijk} A_{\mu \tilde{j}\tilde{k}} X_+^{(I)} + 2f^{jk}{}_i A_{\mu k} X_j^{(I)} - 2\tilde{f}_{ijk} A_{\mu \tilde{k}} X_{\tilde{j}}^{(I)} \\ &\quad + 2\tilde{f}_{ji}{}^k A_{\mu \tilde{j}k} X_{\tilde{+}}^{(I)} + 2f_{ij}{}^k A_{\mu k} X_j^{(I)} + 2\tilde{f}_{ki}{}^j A_{\mu \tilde{k}} X_j^{(I)} \end{aligned} \quad (42)$$

$$\begin{aligned} D_\mu X_{\tilde{i}}^{(I)} &= \partial_\mu X_{\tilde{i}}^{(I)} + f^{ijk} A_{\mu jk} X_+^{(I)} + \tilde{f}_{jk}{}^i A_{\mu \tilde{j}\tilde{k}} X_{\tilde{+}}^{(I)} - 2f^{ijk} A_{\mu k} X_j^{(I)} - \tilde{f}_{jk}{}^i A_{\mu \tilde{k}} X_{\tilde{j}}^{(I)} \\ &\quad + 2f^{ki}{}_j A_{\mu \tilde{j}k} X_{\tilde{+}}^{(I)} - f^{ki}{}_j A_{\mu k} X_{\tilde{j}}^{(I)} - 2f^{ij}{}_k A_{\mu \tilde{k}} X_j^{(I)}, \end{aligned} \quad (43)$$

we will have

$$\begin{aligned} D_\mu X_A^{(I)} D^\mu X^{A(I)} &= \partial_\mu X_-^{(I)} \partial^\mu X^{-(I)} + \partial_\mu X_-^{(I)} \partial^\mu X^{\tilde{-}(I)} + \partial_\mu X_+^{(I)} \partial^\mu X^{+(I)} + \partial_\mu X_{\tilde{+}}^{(I)} \partial^\mu X^{\tilde{+}(I)} \\ &\quad + D_\mu X_i^{(I)} D^\mu X^{i(I)} + D_\mu X_{\tilde{i}}^{(I)} D^\mu X^{\tilde{i}(I)} \end{aligned} \quad (44)$$

where

$$\begin{aligned} D_\mu X_i^{(I)} D^\mu X^{i(I)} &= \partial_\mu X_i^{(I)} \partial^\mu X^{i(I)} + [f^{jk}{}_i A_{\mu jk} X_+^{(I)} + 2\tilde{f}_{ji}{}^k A_{\mu \tilde{j}k} X_{\tilde{+}}^{(I)} + \tilde{f}_{ijk} A_{\mu \tilde{j}\tilde{k}} X_{\tilde{+}}^{(I)} - 2f^{jk}{}_i A_{\mu k} X_j^{(I)} \\ &\quad - 2\tilde{f}_{ijk} A_{\mu \tilde{k}} X_{\tilde{j}}^{(I)} + 2f_{ij}{}^k A_{\mu k} X_j^{(I)} + 2\tilde{f}_{ki}{}^j A_{\mu \tilde{k}} X_j^{(I)}][f^{ijk} A_{\mu jk} X_-^{(I)} - \tilde{f}_{jk}{}^i A_{\mu \tilde{j}\tilde{k}} X_-^{(I)} \\ &\quad - 2f^{ik}{}_j A_{\mu k} X_j^{(I)} + \tilde{f}_{jk}{}^i A_{\mu \tilde{k}} X_{\tilde{j}}^{(I)} + f^{ij}{}_k A_{\mu \tilde{k}} X_j^{(I)} - f^{ikj} A_{\mu k} X_{\tilde{j}}^{(I)} - 2f^{ijk} A_{\mu \tilde{j}\tilde{k}} X_-^{(I)}], \end{aligned} \quad (45)$$

and

$$\begin{aligned}
D_\mu X_{\tilde{i}}^{(I)} D^\mu X^{\tilde{i}(I)} &= \partial_\mu X_{\tilde{i}}^{(I)} \partial^\mu X^{\tilde{i}(I)} + [f^{ijk} A_{\mu jk} X_+^{(I)} + \tilde{f}_{jk}{}^i A_{\mu \tilde{j}\tilde{k}} X_+^{(I)} + 2f^{ki}{}_j A_{\mu \tilde{j}k} X_+^{(I)} - 2f^{ijk} A_{\mu k} X_j^{(I)} \\
&\quad - \tilde{f}_{jk}{}^i A_{\mu \tilde{k}} X_{\tilde{j}}^{(I)} - f^{ki}{}_j A_{\mu k} X_j^{(I)} - 2f^{ij}{}_k A_{\mu \tilde{k}} X_j^{(I)}][2\tilde{f}_{ik}{}^j A_{\mu \tilde{j}\tilde{k}} X_-^{(I)} - 2f^{jk}{}_i A_{\mu jk} X_-^{(I)} \\
&\quad - \tilde{f}_{ijk} A_{\mu \tilde{j}\tilde{k}} X_-^{(I)} - 2f_{ikj} A_{\mu \tilde{k}} X_j^{(I)} - 2\tilde{f}_{ji}{}^k A_{\mu k} X_j^{(I)} + 2f^{jk}{}_i A_{\mu k} X^{\tilde{j}(I)} + 2\tilde{f}_{ik}{}^j A_{\mu \tilde{k}} X^{\tilde{j}(I)}]. \quad (46)
\end{aligned}$$

Furthermore, the first term of CS term in BLG action (14) turns into the following forms:

$$\frac{1}{2} \epsilon^{\mu\nu\lambda} F^{ABCD} A_{\mu AB} \partial_\nu A_{\lambda CD} = 2\epsilon^{\mu\nu\lambda} F^{BCD} A_{\mu BC} \partial_\nu A_{\lambda D} + 2\epsilon^{\mu\nu\lambda} F^{BCD} A_{\mu BC} \partial_\nu A_{\lambda D} \quad (47)$$

where

$$\begin{aligned}
\epsilon^{\mu\nu\lambda} F^{BCD} A_{\mu B} \partial_\nu A_{\lambda CD} &= \frac{1}{3} \epsilon^{\mu\nu\lambda} f^{jk}{}_i A_\mu^i \partial_\nu A_{\lambda jk} + \frac{2}{3} \epsilon^{\mu\nu\lambda} f^{ij}{}_k A_{\mu i} \partial_\nu A_{\lambda j}^k + \frac{2}{3} \epsilon^{\mu\nu\lambda} f^{ki}{}_j A_{\mu i} \partial_\nu A_{\lambda \tilde{j}}^{\tilde{k}} \\
&+ \frac{2}{3} \epsilon^{\mu\nu\lambda} f^{ij}{}_k A_\mu^{\tilde{i}} \partial_\nu A_{\lambda \tilde{j}\tilde{k}} + \frac{2}{3} \epsilon^{\mu\nu\lambda} f^{jk}{}_i A_\mu^{\tilde{i}} \partial_\nu A_{\lambda j}^{\tilde{k}} + \frac{1}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{jk}{}^i A_\mu^{\tilde{i}} \partial_\nu A_{\lambda \tilde{j}\tilde{k}} + \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{ik}{}^j A_\mu^i \partial_\nu A_{\lambda j\tilde{k}} \\
&+ \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{jk}{}^i A_\mu^i \partial_\nu A_{\lambda \tilde{k}}^j + \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{ki}{}^j A_{\mu \tilde{i}} \partial_\nu A_{\lambda j}^k + \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{ij}{}^k A_{\mu \tilde{i}} \partial_\nu A_{\lambda \tilde{j}}^{\tilde{k}} \quad (48)
\end{aligned}$$

and

$$\begin{aligned}
\epsilon^{\mu\nu\lambda} F^{BCD} A_{\mu B} \partial_\nu A_{\lambda CD} &= \frac{2}{3} \epsilon^{\mu\nu\lambda} f^{ijk} A_{\mu i} \partial_\nu A_{\lambda j}^{\tilde{k}} + \frac{1}{3} \epsilon^{\mu\nu\lambda} f^{jki}{}_i A_\mu^{\tilde{i}} \partial_\nu A_{\lambda jk} \\
&+ \frac{1}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{ijk}{}^i A_\mu^i \partial_\nu A_{\lambda \tilde{j}\tilde{k}} + \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{jik}{}^i A_{\mu \tilde{i}} \partial_\nu A_{\lambda \tilde{k}}^j \quad (49)
\end{aligned}$$

and the second term of CS term in BLG action

$$\frac{1}{3} \epsilon^{\mu\nu\lambda} F^{AEF}{}_G F^{BCDG} A_{\mu AB} A_{\nu CD} A_{\lambda EF} = -2\epsilon^{\mu\nu\lambda} F^{ABC} F^{EF}{}_A A_{\mu EF} A_{\nu B} A_{\lambda C} - 2\epsilon^{\mu\nu\lambda} F^{ABC} F^{\mathcal{E}\mathcal{F}}{}_A A_{\mu \mathcal{E}\mathcal{F}} A_{\nu B} A_{\lambda C} \quad (50)$$

where

$$\begin{aligned}
\epsilon^{\mu\nu\lambda} F^{ABC} F^{EF}{}_A A_{\mu EF} A_{\nu B} A_{\lambda C} &= \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i f^{il}{}_m A_{\mu jk} A_{\nu l} A_\lambda^m + \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{ij}{}_k f^{lm}{}_i A_{\mu j}^k A_{\nu l} A_{\lambda m} \\
&+ \epsilon^{\mu\nu\lambda} f^{ij}{}_k \tilde{f}_{mi}{}^l A_{\mu jk} A_{\nu l} A_\lambda^m + \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i \tilde{f}_{lm}{}^i A_{\mu jk} A_{\nu l} A_\lambda^m + \epsilon^{\mu\nu\lambda} f^{ij}{}_k \tilde{f}_{il}{}^m A_{\mu j}^k A_{\nu l} A_{\lambda m} + \epsilon^{\mu\nu\lambda} f^{ij}{}_k f^{lm}{}_i A_{\mu jk} A_{\nu l} A_{\lambda m} \\
&+ \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i f^{mi}{}_l A_{\mu j}^k A_{\nu l} A_{\lambda m} + \epsilon^{\mu\nu\lambda} f^{mi}{}_l \tilde{f}_{ki}{}^j A_{\mu jk} A_{\nu l} A_{\lambda m} + \epsilon^{\mu\nu\lambda} f^{lm}{}_i \tilde{f}_{kj}{}^i A_{\mu k} A_{\nu l} A_{\lambda m} + \frac{1}{2} \epsilon^{\mu\nu\lambda} \tilde{f}_{jk}{}^i \tilde{f}_{mi}{}^l A_{\mu jk} A_{\nu l} A_\lambda^m \\
&+ \epsilon^{\mu\nu\lambda} \tilde{f}_{ki}{}^j \tilde{f}_{lm}{}^i A_{\mu jk} A_{\nu l} A_\lambda^m + \epsilon^{\mu\nu\lambda} \tilde{f}_{kj}{}^i \tilde{f}_{il}{}^m A_{\mu k} A_{\nu l} A_{\lambda m} + \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{im}{}_l \tilde{f}_{ij}{}^k A_{\mu j}^k A_{\nu l} A_{\lambda m} + \frac{1}{4} \epsilon^{\mu\nu\lambda} f^{lm}{}_i \tilde{f}_{jk}{}^i A_{\mu jk} A_{\nu l} A_\lambda^m \\
&+ \epsilon^{\mu\nu\lambda} f^{mi}{}_l \tilde{f}_{ki}{}^j A_{\mu jk} A_{\nu l} A_\lambda^m + \frac{1}{2} \epsilon^{\mu\nu\lambda} \tilde{f}_{jk}{}^i \tilde{f}_{mi}{}^l A_{\mu jk} A_{\nu l} A_{\lambda m} + \frac{1}{2} \epsilon^{\mu\nu\lambda} \tilde{f}_{ij}{}^k \tilde{f}_{lm}{}^i A_{\mu j}^k A_{\nu l} A_{\lambda m} + \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i \tilde{f}_{li}{}^m A_{\mu j}^k A_{\nu l} A_{\lambda m} \\
&+ \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i f^{mi}{}_l A_{\mu jk} A_{\nu l} A_\lambda^m + \epsilon^{\mu\nu\lambda} f^{ij}{}_k \tilde{f}_{il}{}^m A_{\mu jk} A_{\nu l} A_\lambda^m + \frac{1}{2} \epsilon^{\mu\nu\lambda} f^{jk}{}_i \tilde{f}_{lm}{}^i A_{\mu jk} A_{\nu l} A_{\lambda m}, \quad (51)
\end{aligned}$$

and

$$\begin{aligned}
\epsilon^{\mu\nu\lambda} F^{ABC} F^{\mathcal{E}\mathcal{F}}{}_A A_{\mu \mathcal{E}\mathcal{F}} A_{\nu B} A_{\lambda C} &= \frac{1}{3} \epsilon^{\mu\nu\lambda} (-[2f^{jk}{}_i f^{il}{}_m + \frac{3}{2} f^{jki} \tilde{f}_{mi}{}^l + 3f^{lji} \tilde{f}_{mi}{}^k] A_{\mu jk} A_{\nu l} A_\lambda^m \\
&+ [2f^{kj}{}_i f^{li}{}_m - 3f^{lki} \tilde{f}_{mi}{}^j - 2f^{jil} \tilde{f}_{mi}{}^k] A_{\mu \tilde{m}} A_{\nu k}^{\tilde{j}} A_{\lambda l} - [f^{jk}{}_i f^{im}{}_l - 2f^{ijm} \tilde{f}_{il}{}^k + \tilde{f}_{il}{}^m f^{jki}] A_{\mu}^{\tilde{m}} A_{\nu jk} A_{\lambda \tilde{l}} \\
&- 2[2f^{kj}{}_i \tilde{f}_{li}{}^m - \frac{1}{2} f^{ikm} \tilde{f}_{il}{}^j + 2f^{im}{}_l \tilde{f}_{ij}{}^k - \frac{1}{2} f^{mk}{}_i \tilde{f}_{lj}{}^i - \frac{1}{2} f^{kji} \tilde{f}_{ij}{}^m - \frac{1}{2} f^{kim} \tilde{f}_{il}{}^j] A_{\mu}^{\tilde{m}} A_{\nu jk} A_{\lambda \tilde{l}} + 3f_{il}^{\tilde{j}} \tilde{f}_{kij} A_{\mu k} A_{\nu j} A_{\lambda \tilde{m}} \\
&+ [f^{jki} \tilde{f}_{im}{}^l + f^{lki} \tilde{f}_{mi}{}^j + 2f_{ij}^{\tilde{j}} f_{lm}^{\tilde{i}}] A_{\mu k}^{\tilde{j}} A_{\nu}^{\tilde{i}} A_{\lambda l} - [\tilde{f}_{jk}{}^i \tilde{f}_{il}{}^m + \tilde{f}_{mk}{}^i \tilde{f}_{ilm}] A_{\mu j}^{\tilde{k}} A_{\nu \tilde{j}}^{\tilde{i}} A_{\lambda l} + \tilde{f}_{jik} f_{ilm}^{\tilde{j}} A_{\mu m}^{\tilde{k}} A_{\nu}^{\tilde{i}} A_{\lambda l} \\
&- [2\tilde{f}_{ml}{}^i f^{jk}{}_i + 2\tilde{f}_{il}{}^k f^{ij}{}_m + \frac{1}{2} \tilde{f}_{mil} f^{jki} + 2\tilde{f}_{il}{}^k f^{ij}{}_m + \tilde{f}_{mi}{}^j f^{kji} + \tilde{f}_{lm}{}^i f^{jk}{}_i] A_{\mu}^{\tilde{m}} A_{\nu jk} A_{\lambda \tilde{l}} + \tilde{f}_{im}{}^l f^{jji} A_{\mu m}^{\tilde{k}} A_{\nu}^{\tilde{i}} A_{\lambda l} \\
&+ [f^{lmi} f^{jk}{}_i - f^{jli} f^{mk}{}_i - 2f^{imk} f^{jl}{}_i + 2f^{ijk} f^{lm}{}_i \tilde{f}_{mi}{}^l f^{ijk} - \frac{1}{2} f^{lmi} f^{kj}{}_i] A_{\mu}^{\tilde{k}} A_{\nu j} A_{\lambda lm} + \tilde{f}_{ijk} f^{lim} A_{\mu}^{\tilde{m}} A_{\nu \tilde{j}\tilde{k}} A_{\lambda \tilde{m}}^{\tilde{l}}
\end{aligned}$$

$$\begin{aligned}
& -2[f_j^{ki} f_i^{lm} + f_j^{mk} f_i^{il} A_{\mu\tilde{m}}^{\tilde{k}} A_{\nu j} A_{\lambda l} + \frac{1}{2} \tilde{f}_{ji}^k f^{mli} + \frac{1}{2} \tilde{f}_{li}^m f^{ikj} - \tilde{f}_{ji}^l f^{mik}] A_{\mu m} A_{\nu\tilde{j}}^{\tilde{k}} A_{\lambda l} + 2f_j^{ki} f_i^{lm} A_{\mu km} A_{\nu\tilde{j}} A_{\lambda\tilde{l}} \\
& -[f_j^{ml} \tilde{f}_{jk}^i - f_j^{mi} \tilde{f}_{jk}^l + 2f_{il}^k \tilde{f}_{ij}^m] A_{\mu}^{\tilde{m}} A_{\nu\tilde{j}\tilde{k}} A_{\lambda l} + 3[-\frac{1}{2} \tilde{f}_{jik} \tilde{f}_{lm}^i + \frac{2}{3} \epsilon^{\mu\nu\lambda} \tilde{f}_{kl}^i \tilde{f}_{jim}] A_{\mu\tilde{k}} A_{\nu}^j A_{\lambda\tilde{l}\tilde{m}} - \tilde{f}_{ik}^j f_{lm}^i A_{\mu}^{\tilde{k}} A_{\nu}^j A_{\lambda\tilde{l}\tilde{m}} \\
& +[\tilde{f}_{jk}^i \tilde{f}_{im}^l + 2\tilde{f}_{ji}^l \tilde{f}_{km}^i + 2\tilde{f}_{mik} f_{il}^j + \tilde{f}_{ijk} f_{lm}^i] A_{\mu}^m A_{\nu\tilde{j}\tilde{k}} A_{\lambda l} + [\tilde{f}_{ml}^i f^{jk} - \tilde{f}_{li}^k f^{ij} - 2f_{il}^j \tilde{f}_{im}^k - 2f^{ijk} \tilde{f}_{mli}] A_{\mu k} A_{\nu j} A_{\lambda\tilde{l}}^m \\
& -[3\tilde{f}_{ik}^j \tilde{f}_{ml}^i + \tilde{f}_{im}^j \tilde{f}_{kli} + 2\tilde{f}_{ilk} f_{jm}^i] A_{\mu\tilde{k}\tilde{m}} A_{\nu}^j A_{\lambda\tilde{l}} - 2[2\tilde{f}_{ij}^k \tilde{f}_{ml}^i - f_{lm}^{ki} \tilde{f}_{ilj} A_{\mu\tilde{m}} - f_j^{ki} \tilde{f}_{ilm} A_{\mu\tilde{m}}^{\tilde{k}} - \frac{1}{2} \tilde{f}_{mi}^l f_{il}^{ik}] A_{\mu\tilde{m}} A_{\nu\tilde{j}} A_{\lambda\tilde{l}} \\
& -[2\tilde{f}_{ij}^k f_{il}^{im} - 3f_j^{ki} \tilde{f}_{li}^m + 2\tilde{f}_{il}^k f_j^{im} + \tilde{f}_{lk}^i f_i^{mj}] A_{\mu m} A_{\nu\tilde{j}}^{\tilde{k}} A_{\lambda\tilde{l}} [\tilde{f}_{im}^l \tilde{f}_{jk}^i + 2f_{il}^j \tilde{f}_{kim} + 3\tilde{f}_{mj}^i \tilde{f}_{ik}^l + 2f_{il}^{jj} \tilde{f}_{km}^i] A_{\mu\tilde{m}} A_{\nu\tilde{j}}^{\tilde{k}} A_{\lambda\tilde{l}} \\
& -[2f_{ij}^{ij} \tilde{f}_{kl}^m - 2f_{jm}^i \tilde{f}_{kl}^i - 2f_{ik}^m \tilde{f}_{jl}^i + f_{ik}^{im} \tilde{f}_{ijl} - f_{kl}^{kj} f_{jm}^i + 2f_{il}^j \tilde{f}_{ki}^m - f^{mij} \tilde{f}_{kil} + \tilde{f}_{jk}^i \tilde{f}_{mi}^l + f_{jm}^{mi} \tilde{f}_{kil} \\
& + 2f_{il}^{mi} \tilde{f}_{ik}^j] A_{\mu}^k A_{\nu j} A_{\lambda\tilde{l}m} - [2f_{ij}^{ki} f_{lm}^i + \tilde{f}_{ji}^k f_{ilm}^i + 2\tilde{f}_{ji}^l f_{im}^{ik} - 3f_{kil}^i \tilde{f}_{ij}^m] A_{\mu}^{\tilde{m}} A_{\nu\tilde{j}\tilde{k}} A_{\lambda l} + \tilde{f}_{ki}^j f_{lm}^{im} A_{\mu m} A_{\nu j}^k A_{\lambda l} \\
& -[4\tilde{f}_{ij}^k \tilde{f}_{il}^i - f_{il}^{ik} \tilde{f}_{im}^j + f_{im}^{ik} \tilde{f}_{ilj} A_{\mu k\tilde{m}}] A_{\mu\tilde{m}} A_{\nu\tilde{j}\tilde{k}} A_{\lambda}^l + 2[\tilde{f}_{jk}^i \tilde{f}_{il}^m + \tilde{f}_{ij}^m \tilde{f}_{kl}^i] A_{\mu\tilde{k}m} A_{\nu\tilde{j}} A_{\lambda}^l + [2\tilde{f}_{jk}^i f_{lm}^i + f_{jm}^{im} \tilde{f}_{kl}^l \\
& - 2\tilde{f}_{jki} \tilde{f}_{lim} + f_{kl}^{li} \tilde{f}_{ij}^m - f_{ik}^{im} \tilde{f}_{ij}^l] A_{\mu\tilde{k}} A_{\nu\tilde{j}} A_{\lambda\tilde{l}}^{\tilde{m}} - [2f_{lm}^{li} \tilde{f}_{ij}^k + \tilde{f}_{ijl} f_{lm}^{ki} - f_{lm}^{ik} \tilde{f}_{ij}^l - f_{ij}^{ik} \tilde{f}_{lm}^l - f_{jl}^{li} \tilde{f}_{mi}^k] A_{\mu k} A_{\nu j} A_{\lambda\tilde{l}}^m), \quad (52)
\end{aligned}$$

where F^{ABC} is the structure constant of the Manin triple of Lie bialgebra $(\mathcal{D}, \mathcal{G}, \mathcal{G}^*)^7$. Note that indices A and \mathcal{A} can be i and \tilde{i} . In the above relations we have used the notations $A_{\mu+B} = A_{\mu\tilde{+}B} = A_{\mu B}$, $A_{\mu+\tilde{B}} = A_{\mu\tilde{+}\tilde{B}} = A_{\mu\tilde{B}}$, $F^{AB}{}_C A_{\mu AB} \equiv C_{\mu C}$ and $F^{AB}{}_C A_{\mu AB} \equiv \tilde{C}_{\mu C}$ that $\mathcal{A} = i, \tilde{i}$ and they can't choose only from \mathcal{G} or \mathcal{G}^* , then the sum of (47) and (50) will have the following form:

$$\frac{1}{2} \epsilon^{\mu\nu\lambda} \{C_{\mu B} (\partial_\nu A_\lambda^B - \partial_\lambda A_\nu^B - [A_\nu, A_\lambda]_B) + \tilde{C}_\mu^B (\partial_\nu A_{\lambda B} - \partial_\lambda A_{\nu B} - [A_\nu, A_\lambda]_B)\}. \quad (55)$$

In this way the general form of the BLG Lagrangian on the especial 3-Lie algebra (Manin triple) (33) is as follows ⁸:

$$L = \frac{1}{2} D_\mu X^{A(I)} D^\mu X_A^{(I)} - 2g_{YM}^2 C_\mu^B C_B^\mu - 2g_{YM} \tilde{C}_{\mu B} D^\mu X^{(8)B} + 2\epsilon^{\mu\nu\lambda} C_{\mu A} B_{\nu\lambda}^A + 2\epsilon^{\mu\nu\lambda} \tilde{C}_\mu^B F_{\nu\lambda B} + E \quad (56)$$

$$\begin{aligned}
E &= g_{YM} C^{\mu i} \partial_\mu X_i^{(I)} + g_{YM} C_{\mu i} \partial^\mu X^{i(I)} + C_{\mu i} \tilde{C}^{\mu i} - 2g_{YM} f^{ikj} A_{\mu k} X^{j(I)} C_{\mu i} + g_{YM} \tilde{f}_{jk}^i A_{\mu\tilde{k}} C_{\mu i} \\
&+ g_{YM} f_{jk}^{ij} A_{\mu\tilde{k}} X^{\tilde{j}(I)} C_{\mu i} + g_{YM} f^{ikj} A_{\mu k} X^{\tilde{j}(I)} C_{\mu i} + \dots
\end{aligned} \quad (57)$$

where

$$B_{\nu\lambda A} = \partial_\nu A_{\lambda A} - \partial_\lambda A_{\nu A} - [A_\nu, A_\lambda]_A, \quad (58)$$

$$F_{\nu\lambda}^A = \partial_\nu A_\lambda^A - \partial_\lambda A_\nu^A - [A_\nu, A_\lambda]^A. \quad (59)$$

Now, by integration of $C_{\mu k}$ and \tilde{C}_μ^k

$$C_{\mu A} = \frac{1}{g_{YM}^2} \epsilon_\mu^{\nu\lambda} B_{\nu\lambda A} + \frac{1}{g_{YM}} \partial_\mu X_A^{(I)} + \dots \quad (60)$$

and

$$\tilde{C}_{\mu A} = \frac{1}{g_{YM}^2} \epsilon_\mu^{\nu\lambda} F_{\nu\lambda A} + \frac{1}{g_{YM}} \partial_\mu X_A^{(I)} + \dots \quad (61)$$

⁷Note that 3-Lie bialgebra isn't direct sum of \mathcal{A} and \mathcal{A}^* if so, we couldn't have relation $[T^+, T^i, T^{\tilde{j}}] = f^{ikj} T^{\tilde{k}}$ and $[T^{\tilde{+}}, T^{\tilde{i}}, T^{\tilde{j}}] = \tilde{f}_{ik}^j T^k$ and proposition in the previous section will fail for this case. Now, we investigate this case i.e. direct sum for our model then relation (48) turn into following form:

$$\epsilon^{\mu\nu\lambda} F^{BCD} A_{\mu B} \partial_\nu A_{\lambda CD} = f^{ij}{}_k A_{\mu i} \partial_\nu A_{\lambda j}^k + \tilde{f}_{ij}^k A_{\mu i} \partial_\nu A_{\lambda\tilde{j}}^{\tilde{k}} \quad (53)$$

and relation (51) as follows:

$$\epsilon^{\mu\nu\lambda} F^{ABC} F^{EF}{}_A A_{\mu EF} A_{\nu B} A_{\lambda\tilde{C}} = \epsilon^{\mu\nu\lambda} f^{jk}{}_i f_{lm}^{il} A_{\mu l}^m A_{\nu j} A_{\lambda k} + \epsilon^{\mu\nu\lambda} \tilde{f}_{ki}^j \tilde{f}_{ml}^i A_{\mu\tilde{m}}^l A_{\nu j} A_{\lambda\tilde{k}} \quad (54)$$

the result is the same one in Ref. [14] with one difference that we will have two Yang-Mills action one for \mathcal{A} and the other for \mathcal{A}^* .

⁸Note that in this Lagrangian the E term (also "+" terms) according to relations (44-46), can not contribute in Yang-Mills and DBI actions.

then by insertion in the Lagrangian we will obtain the following equation:

$$L = \frac{1}{2}F_{\nu\lambda A}F^{\nu\lambda A} + \frac{1}{2}B_{\nu\lambda A}B^{\nu\lambda A} + \frac{1}{2}D_\mu X^{A(I)}D^\mu X_A^{(I)} + \dots, \quad (62)$$

where $F_{\nu\lambda A}$ is field strength of Yang-Mills (with the gauge field $A_{\lambda A}$) and $B_{\nu\lambda A}$ is related to B-field of a string. As we know, the dynamic of D-branes where are expressed by DBI and Yang-Mills action can be obtained by expanding DBI action [35, 36]. Furthermore, relation between DBI action and sigma model have been investigated by Leigh in Ref. [37] so the relation between DBI action and WZW models that are sigma model on Lie groups can be exist. In this way, our claim might be true that expanding of DBI action have a term that correspond to a B-field of a WZW model. We must consider that fields are 3-Lie algebraic valued in this WZW model that we will obtain its form in the next subsection. Therefore, we will have WZW model from BLG model which have been constructed on Manin triple otherwise we will have Yang-Mills model from it [14]. In this way, we provide a method to obtain D2 from M2 and vice versa. Note that this method for obtaining D2 from M2 is different from method of Ref. [12]. In the mentioned reference one can not obtain M2 from D2 but in our method this is possible. The BLG model is maximally supersymmetric ($N = 8$) in $2 + 1$ dimension, with correlating DBI action being $N = (4, 4)$ supersymmetric or a string $N = (4, 4)$ supersymmetric. We know that if string propagates on a group manifold, one can replace the string action with WZW action. Therefore, if we assume that our string model propagates on Lie group, then we will have an $N = (4, 4)$ supersymmetric WZW model in two dimension. In the previous work we analyzed the algebraic structure of $N = (4, 4)$ supersymmetric WZW model and showed that this model has Lie bialgebra structure with one 2-cocycle [26]. Therefore, Lie algebra \mathcal{G} in (29) must have a Lie bialgebraic structure with 2-cocycles. In this way by starting with an $N = (4, 4)$ WZW model (D2-model) with Lie algebra \mathcal{G} (where it is a Lie bialgebra with one 2-cocycle) one can obtain a BLG model (M2-model) by 3-Lie algebra with commutation relation (33) which is obtained from \mathcal{G} and its dual \mathcal{G}^* . Note that, contrary to the ordinary WZW model, here in this model the B-field has algebraic index and therefore we have $N = (4, 4)$ like WZW model (the form of B-field has been shown in (58)).

4.1 WZW model with 3-Lie algebra valued fields

We know that the WZW action has following form [38]:

$$S_{WZW} = \int d^3x \epsilon^{\alpha\beta\gamma} L_\mu^I L_\nu^J L_\lambda^K \partial_\alpha X^\mu \partial_\beta X^\nu \partial_\gamma X^\lambda \text{Tr}([T_I, T_J], T_K) \quad (63)$$

Now by setting the algebraic index for space-time coordinate ($X^\mu = X^{\mu A} T_A$) one can write the WZW-like term with the following form:

$$S_{WZW-like} = \int d^3x \epsilon^{\alpha\beta\gamma} L_\mu^L L_\nu^M L_\lambda^N \partial_\alpha X^{I\mu} \partial_\beta X^{J\nu} \partial_\gamma X^{K\lambda} \text{Tr}([T_I T_L, T_J T_M], T_K T_N), \quad (64)$$

subsequently, we anticipate that the B-field of the WZW-like model have two algebraic indices as the following form:

$$\begin{aligned} S_{WZW-like} &= \int d^2x \left\{ \frac{1}{6} \epsilon^{\beta\gamma} B_{\nu\mu}^Q \partial_\beta X^{J\nu} \partial_\gamma X^{I\mu} \text{Tr}(T_J T_I T_Q) + \frac{1}{6} \epsilon^{\alpha\gamma} B_{\nu\lambda}^Q \partial_\alpha X^{J\nu} \partial_\gamma X^{K\lambda} \text{Tr}(T_J T_K T_Q) \right. \\ &\quad \left. + \frac{1}{6} \epsilon^{\alpha\gamma} B_{\mu\lambda}^Q \partial_\alpha X^{I\mu} \partial_\gamma X^{K\lambda} \text{Tr}(T_I T_K T_Q) \right\} + \dots, \end{aligned} \quad (65)$$

where $B_{\nu\mu}^Q = L_\nu^L L_\mu^N f_{NL}^P x^J f_{PJ}^Q$, $L_\mu^L X^{I\mu} T_I T_L|_{\text{boundary}} = x^L T_L|_{\text{boundary}}$ such that the $B_{\nu\mu}^Q$ have the form of 58, in this way the kinetic term for WZW-like action has the following form:

$$\int L_\mu^L L_\nu^M \partial X^{I\mu} \partial X^{J\nu} f_{LM}^Q \text{Tr}(T_Q T_I T_J). \quad (66)$$

Therefore, we see that the second term of (62) is related to the WZW-like action, i.e., a $N = (4, 4)$ string (D2-model) with the Lie algebra \mathcal{G} as a Manin triple of Lie bialgebra with one 2-cocycle. This means that if we have an $N = (4, 4)$ WZW model (D2-model) with Lie algebra \mathcal{G} (as a Manin triple of a Lie bialgebra with one 2-cocycle) we will have a BLG model (M2-model) related to 3-Lie algebra [35] (which is obtained from \mathcal{G} and \mathcal{G}^*) and vice versa.

Conclusions

Using the concept of 3-Lie bialgebra (recently defined in arXiv:1604.04475) we have constructed BLG model on a Manin triple of a especial 3-Lie bialgebra $(\mathcal{D}, \mathcal{A}, \mathcal{A}^*)$. Then, using the correspondence between 3-Lie bialgebra $(\mathcal{A}, \mathcal{A}^*)$ and Lie bialgebra $(\mathcal{G}, \mathcal{G}^*)$ we have shown that the BLG model can be reduce to an $N = (4, 4)$ WZW model on a Lie algebra \mathcal{G} such that the Lie algebra had a one 2-cocycle.

In this way one can begin with a D2-models with Lie algebra \mathcal{G} and construst M2-model over \mathcal{D} and vice versa. One of the open problems is that one can classify such M2-models by using the classification of Lie bialgebras \mathcal{G} .

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Appendix

The adjoint representation of the 3-Lie algebra and ad-invariant metric on \mathcal{D}

Here we assume that \mathcal{D} of Manin triple $(\mathcal{D}, \mathcal{A}, \mathcal{A}^*)$ is a 3-Lie algebra.

A Manin triple $(\mathcal{D}, \mathcal{A}, \mathcal{A}^*)$ which is in one-to-one correspondence with 3-Lie bialgebras $(\mathcal{A}, \mathcal{A}^*)$ must have a nondegenerate ad-invariant inner product over 3-Lie algebra \mathcal{D} . In order to calculate this we choose the basis of $\mathcal{D} = \mathcal{A} \oplus \mathcal{A}^*$ as; $T^A(\{T^-, T^+, T^i, \tilde{T}_-, \tilde{T}_+, \tilde{T}_i\}) = \{T^-, T^+, T^i, T^{\bar{-}}, T^{\bar{+}}, T^{\bar{i}}\}$ and use the commutation relation on \mathcal{A} and \mathcal{A}^* :

$$\begin{aligned} [T^-, T^a, T^b] &= 0, & [T^+, T^i, T^j] &= f^{ij}{}_k T^k, & [T^i, T^j, T^k] &= f^{ijk} T^-, \\ [\tilde{T}_-, \tilde{T}_a, \tilde{T}_b] &= 0, & [\tilde{T}_+, \tilde{T}_i, \tilde{T}_j] &= \tilde{f}_{ij}{}^k \tilde{T}_k, & [\tilde{T}_i, \tilde{T}_j, \tilde{T}_k] &= \tilde{f}_{ijk} T^-, \end{aligned}$$

Now, commutation relation for the 3-Lie algebras \mathcal{D} have the following form:

$$[T^A, T^B, T^C] = F^{ABC}{}_D T^D. \quad (67)$$

which we use for simplification:

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{pmatrix}, \quad (68)$$

where a, c, i, k are 2×2 matrices, b, d, j, l are $2 \times n$ matrices, e, g, m, p are $n \times 2$ matrices and f, h, n, q are $n \times n$ matrices. Then, the adjoint representation of basis of 3-Lie algebra \mathcal{D} have the following form:

$$\begin{aligned} (\mathcal{Y}^{ij})^c{}_d &= \begin{pmatrix} 0 & b_1 & 0 & 0 \\ e_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_1 \\ 0 & 0 & p_1 & 0 \end{pmatrix}, & (\mathcal{Y}^{\bar{i}\bar{j}})^c{}_d &= \begin{pmatrix} 0 & b_2 & 0 & 0 \\ e_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_2 \\ 0 & 0 & p_2 & 0 \end{pmatrix}, \\ (\mathcal{Y}^{+i})^c{}_d &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_1 \end{pmatrix}, & (\mathcal{Y}^{\bar{+}\bar{i}})^c{}_d &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & q_2 \end{pmatrix}, \end{aligned} \quad (69)$$

and we have, $\mathcal{Y}^{-i} = 0$, $\mathcal{Y}^{-\tilde{i}} = 0$, $\mathcal{Y}^{-A} = 0$, $\mathcal{Y}^{-\tilde{A}} = 0$, where

$$\begin{aligned}
b_1 &= \begin{pmatrix} 0 & \cdots & 0 \\ f^{ij}_1 & \cdots & f^{ij}_n \end{pmatrix}, e_1 = \begin{pmatrix} f^{ij1} & 0 \\ \vdots & \vdots \\ f^{ijn} & 0 \end{pmatrix}, l_1 = \begin{pmatrix} f^{ij1} & \cdots & f^{ijn} \\ 0 & \cdots & 0 \end{pmatrix}, p_1 = \begin{pmatrix} 0 & f^{ij}_1 \\ \vdots & \vdots \\ 0 & f^{ij}_n \end{pmatrix}, \\
b_2 &= \begin{pmatrix} \tilde{f}^{ij1} & \cdots & \tilde{f}^{ijn} \\ 0 & \cdots & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & \tilde{f}^{ij1} \\ \vdots & \vdots \\ 0 & \tilde{f}^{ijn} \end{pmatrix}, l_2 = \begin{pmatrix} 0 & \cdots & 0 \\ \tilde{f}^{ij1} & \cdots & \tilde{f}^{ijn} \end{pmatrix}, p_2 = \begin{pmatrix} \tilde{f}^{ij1} & 0 \\ \vdots & \vdots \\ \tilde{f}^{ijn} & 0 \end{pmatrix}, \\
f_1 &= \begin{pmatrix} f^{i1}_1 & \cdots & f^{i1}_n \\ \vdots & \vdots & \vdots \\ f^{in}_1 & \cdots & f^{in}_n \end{pmatrix}, q_1 = \begin{pmatrix} f^{i1}_1 & \cdots & f^{in}_1 \\ \vdots & \vdots & \vdots \\ f^{i1}_n & \cdots & f^{in}_n \end{pmatrix}, \\
f_2 &= \begin{pmatrix} \tilde{f}^{i1}_1 & \cdots & \tilde{f}^{in}_1 \\ \vdots & \vdots & \vdots \\ \tilde{f}^{i1}_n & \cdots & \tilde{f}^{in}_n \end{pmatrix}, q_2 = \begin{pmatrix} \tilde{f}^{i1}_1 & \cdots & \tilde{f}^{in}_1 \\ \vdots & \vdots & \vdots \\ \tilde{f}^{i1}_n & \cdots & \tilde{f}^{in}_n \end{pmatrix}, \tag{70}
\end{aligned}$$

where, $f^{ij}_k = -(\mathcal{X}^i)^j_k$ and $\tilde{f}^{ij}_k = -(\tilde{\mathcal{X}}^i)^j_k$. The non-degenerate ad-invariant inner product for Lie algebras is the result of applying the trace of bilinear product in adjoint representation [39]. However, if the Lie algebra is non-semisimple, current method is not useful. Obtaining the nondegenerate ad-invariant metric for these algebras results from solving the following equation:

$$f^{AB}{}_D G^{CD} = -f^{AC}{}_D G^{DB}. \tag{71}$$

Now, we generalize above result for 3-Lie algebra \mathcal{D} and choose the basis of $\mathcal{D} = \mathcal{A} \oplus \mathcal{A}^*$ as; $T^A(\{T^-, T^+, T^i, \tilde{T}^-, \tilde{T}^+, \tilde{T}^i\}) = \{T^-, T^+, T^i, T^{\tilde{-}}, T^{\tilde{+}}, T^{\tilde{i}}\}$, to obtain the inner product as follows:

$$\begin{aligned}
\langle T^A, [T^B, T^C, T^D]_{\mathcal{D}} \rangle &= -\langle [T^A, T^B, T^C]_{\mathcal{D}}, T^D \rangle \\
F^{BCD}{}_E G^{EA} &= -F^{ABC}{}_E G^{ED}, \tag{72}
\end{aligned}$$

By choosing $(\mathcal{Y}^{AB})^C{}_D = F^{ABC}{}_D$ then (72) means that $\mathcal{Y}^{AB}G$ must be antisymmetric. We have shown their matrix representations as follows and have concluded that the metric must have the following form:

$$G = \begin{pmatrix} a & 0 & b & 0 \\ 0 & 0 & 0 & h \\ i & 0 & k & 0 \\ 0 & n & 0 & 0 \end{pmatrix}, \tag{73}$$

where

$$a = \begin{pmatrix} 0 & g_{11} \\ g_{12} & 0 \end{pmatrix}, b = \begin{pmatrix} 0 & g_{21} \\ g_{22} & 0 \end{pmatrix}, i = \begin{pmatrix} 0 & g_{31} \\ g_{32} & 0 \end{pmatrix}, k = \begin{pmatrix} 0 & g_{41} \\ g_{42} & 0 \end{pmatrix}, \tag{74}$$

in which matrices h and n are arbitrary matrices.

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